Problem 9
Consider a mini spring launcher, using kelp as a spring, that launches a $m = 40$g zucchini at a $\theta = 35°$ angle. If the kelp spring is compressed by $d = 20$cm, the zucchini will hit the floor $\Delta y = 2$m below the point at which it loses contact with the kelp spring. The zucchini travels $\Delta x = 5.2$ m horizontally. What is the spring constant of the kelp?

Solution

We can first solve for the velocity v. We can form a system of equations with two unknowns, t the time zucchini travels before hitting the ground, and v, as indicated in Figure 1.

Rightwards and upwards are $+x$ and $+y$ directions.

$$\Delta x = (v\cos\theta)t \qquad \text{kinematics, } x\text{-direction} \tag{1}$$

$$v = \frac{\Delta x}{t\cos\theta} \tag{2}$$

$$-\Delta y = (v\sin\theta)t - \frac{1}{2}gt^2 \qquad \text{kinematics, } y\text{-direction} \tag{3}$$

$$-\Delta y = \frac{\Delta x}{t\cos\theta}t\sin\theta - \frac{1}{2}gt^2 \qquad \text{from 2} \tag{4}$$

$$\frac{1}{2}gt^2 = \Delta x\tan\theta + \Delta y \tag{5}$$

$$t = \sqrt{\frac{2}{g}(\Delta x\tan\theta + \Delta y)} \tag{6}$$

$$v = \frac{\Delta x}{\sqrt{\frac{2}{g}(\Delta x\tan\theta + \Delta y)}\cos\theta} \qquad \text{from 2 6} \tag{7}$$

$$= \frac{\Delta x}{\cos\theta}\sqrt{\frac{g}{2(\Delta x\tan\theta + \Delta y)}} \tag{8}$$

We can then use a conservation of energy argument to solve for the the spring constant k. Let the vertical position of the zucchini at the very moment it was launched from the kelp spring be the point of zero gravitational potential energy $U_g = 0$ (see Figure 2). We are going to compare energies between the point when the spring is fully compressed, and the point just when the zucchini is launched.

$$E_0 = E_1 \tag{9}$$

$$U_g + U_s = K \tag{10}$$

$$mg(-d\sin\theta) + \frac{1}{2}kd^2 = \frac{1}{2}mv^2 \tag{11}$$

$$kd^2 = mv^2 + 2mgd\sin\theta \tag{12}$$

$$k = \frac{m}{d^2}(v^2 + 2gd\sin\theta) \tag{13}$$

$$= \frac{mg}{d^2}\left(\frac{(\Delta x)^2}{2(\Delta x\tan\theta + \Delta y)\cos^2\theta} + 2d\sin\theta\right) \tag{14}$$

$$= 37.25 \text{ N/m} \tag{15}$$

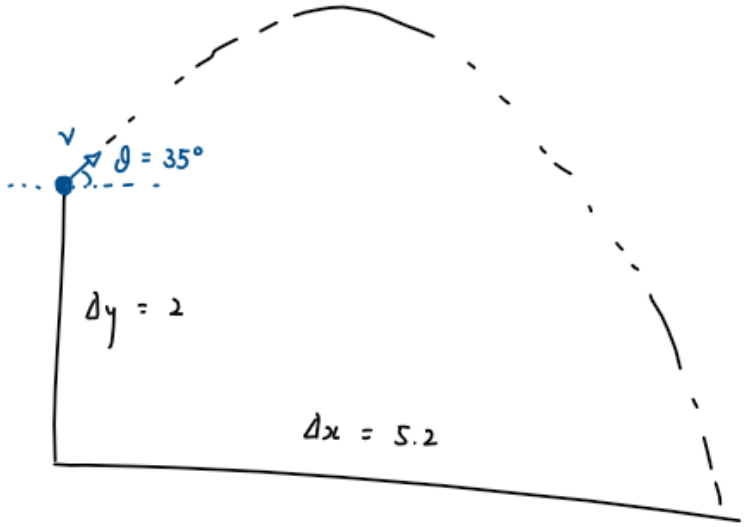

Figure 1: Diagram for kinematics portion of question. We can find velocity the zucchini is launched at.

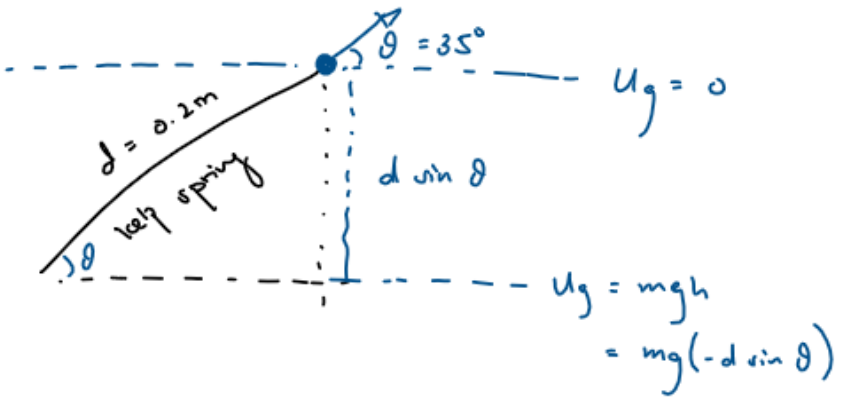

Figure 2: Diagram for energy portion of question. We can use our velocity to find spring constant.